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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The subset selection algorithm is extended to search for a best subset from a large set of complex-valued basis functions. This algorithm is used to design digital finite-duration impulse response (FIR) filters having fewer coefficients than conventional FIR filters. An optimum conventional FIR filter is derived which has best uniform spacing of the fixed number of samples which are to be used, and examples are presented which show that, for the same number of coefficients, the complex-subset-selection filter can give

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better results than the optimum conventional filter.

The complex subset selection method is also applied to estimation of the frequencies of sinusoids in the presence of noise. A windowing technique is introduced to increase the efficiency and accuracy of the algorithm for frequency estimates. The results are compared with Cramer-Rao bounds.

I. Introduction. During the contract period substantial progress was made on three basic problem areas:

(1) The partial basis problem.

(2) Least squares approximation with restricted range.

(3) Mixed norm approximation.

In sections II, III, IV, we will discuss some of our work in each of these areas. In section V some other results are discussed.

II. The Partial Basis Problem. This problem was formulated by us in 1975 as a generalization of a problem of placing antenna elements optimally in a line array; independently, G. G. Lorentz studied a special case [11]. The problem has received considerable attention from researchers in recent years and elegant results have been obtained by us and others. This should prove to be a fruitful area for further investigation.

The partial basis problem can be stated in its general form as follows. Let X be a normed linear space, let $f, h_o, \cdots, h_{N-1}$ belong to X and let n be an integer, $1 \leq n < N$. For every sequence $\mu = \{\mu_k\}_{k=1}^{n}$ of integers, with $0 \leq \mu_1 < \cdots < \mu_n \leq N-1$, consider

$$e(\mu) = \min_{c_1, \cdots, c_n} \left\| f - \sum_{k=1}^{n} c_k h_{\mu_k} \right\| .$$

The problem is to minimize $e(\mu)$; it is of particular interest when X is one of the standard function spaces. The main results of [7] gives sufficient conditions for the "tail" $h_{N-n}, \cdots, h_{N-1}$, to be the unique best partial basis of size n. Typical theorems are:

<u>Theorem 1</u>. Let $0 < a < b < \infty$ and let N,n be integers with $1 \leq n < N$. Let f be a real function, continuous in $[a,b]$ and assume that, for $k = 0, 1, \cdots, n$, $(x^{k-N}f)^{(k)}$ exists and is ≥ 0 in (a,b), with strict inequality there for $k = n-1$ and $k = n$. Let $0 \leq \mu_1 < \cdots < \mu_n \leq N-1$ be integers such that $\{\mu_1, \cdots, \mu_n\} \neq \{N-n, \cdots, N-1\}$. Let $1 \leq p \leq \infty$. Then

$$\min_{c_k \text{ real}} \left\| f(x) - \sum_{k=N-n}^{N-1} c_k x^k \right\|_{L^p(a,b)} <$$

$$\min_{c_k \text{ real}} \left\| f(x) - \sum_{k=1}^{n} c_k x^{\mu_k} \right\|_{L^p(a,b)} .$$

<u>Theorem 2</u>. Let $0 \leq \alpha_{N-1} < \alpha_{N-2} < \cdots < \alpha_0 < \frac{1}{2}$ and let $1 \leq n < N$, n an integer. Let f be a real function with $f^{(2k+1)}(0) = 0$, $k = 0, \cdots, N-1$, and $f^{(2k)}(x) > 0$ on $(0,\pi]$ for $k = 0, 1, \cdots, N$. Assume $f^{(2N-1)}(x)$ is continuous from the right at 0. Let $0 \leq \mu_1 < \cdots < \mu_n \leq N-1$ be integers such that $\{\mu_1, \cdots, \mu_n\} \neq \{N-n, \cdots, N-1\}$. Let $1 \leq p \leq \infty$. Then

$$\min_{c_k \text{ real}} \left\| f(x) - \sum_{k=N-n}^{N-1} c_k \cos \alpha_k x \right\|_{L^p(0,\pi)} <$$

$$\min_{c_k \text{ real}} \left\| f(x) - \sum_{k=1}^{n} c_k \cos \alpha_{\mu_k} x \right\|_{L^p(0,\pi)} .$$

The theory of Tchebycheff systems is important for partial basis results and [7] contains contributions to this theory. A typical theorem:

Theorem 3. Let $0 < \alpha_0 < \cdots < \alpha_n$. Then $\cos \alpha_n x$, $\sin \alpha_0 x$, $-\cos \alpha_1 x$, $-\sin \alpha_1 x, \cdots, (-1)^n \cos \alpha_n x, (-1)^n \sin \alpha_n x$ is an extended complete Tchebycheff system on $[0, \pi]$ if and only if $\alpha_n < 1/2$.

Elegant results have been obtained by Lewis, Pinkus, and Shisha [9] giving sufficient conditions for the "head" $h_0, \cdots, h_{n-1}$ to be the unique best partial basis of size n. Perhaps the most striking result is this:

Theorem 4. Let $0 < a < b < \infty$, $1 \leq n < N$, f continuous and positive on $[a, b]$, $(-1)^k f^{(k)}(x) > 0$ for $a < x < b$, $k = 1, \cdots, n$ and $1 \leq p \leq \infty$. Then for $s = 1, \ldots, n+1$ any subsequence of length s of $f, 1, x, \cdots, x^{N-1}$ is a Tchebycheff system on $[a, b]$ and

$$\min_{c_k \text{ real}} \left\| f(x) - \sum_{k=0}^{n-1} c_k x^k \right\|_{L^p(a,\, b)} < \min_{c_k \text{ real}} \left\| f(x) - \sum_{k=1}^{n} c_k x^{\mu_k} \right\|_{L^p(a,\, b)}$$

whenever $0 \leq \mu_1 < \cdots < \mu_n \leq N-1$ and $\{\mu_1, \cdots, \mu_n\} \neq \{0, 1, \cdots, n-1\}$.

The classical completely monotonic functions satisfy the hypotheses of Theorem 4 and hence are best approximated by $1, x, \cdots, x^{n-1}$. Examples are $f(x) = e^{-cx}$, $c > 0$, $[a, b] \subset (0, \infty)$ and $f(x) = x^{-M}$, M = positive integer, $[a, b] \subset (0, \infty)$.

The problem of finding the best partial basis computationally, for situations where theorems of the above type are inapplicable, was studied in [1]. An algorithm was presented, generalizing an algorithm of Hocking and Leslie [12], and a computer program was written and tested on some numerical examples. Also a theorem givin continuous dependence of a best approximation on the basis functions was given in [1]. This algorithm

was extended to complex-valued basis functions and used to design finite impulse response digital filters and to estimate frequencies of sinusoids in the presence of noise; cf. [4].

III. Least Squares Approximation with Restricted Range. This problem can be formulated as follow:

$$\underset{h \in H}{\text{minimize}} \int_a^b [f(x) - h(x)]^2 dx$$

subject to $\ell(x) \leq p(x) \leq u(x)$ for all x in a set I. This includes the problem of positive approximation (where $p(x) \geq 0$ is required) which has been studied by several investigators ([13], [14]).

The following characterization theorem has been obtained (under certain hypotheses on H, f, ℓ, u, and I which will not be specified here):

The approxi ation h^* in H is a solution of the least squares restricted range problem if and only if $\ell \leq h^* \leq u$ on I and there exists points $x_1, \cdots, x_k$ in I and constants $\sigma_1, \cdots, \sigma_k$ such that:

(i) For $i = 1, \cdots, k$, either $h^*(x_i) = \ell(x_i)$ or $h^*(x_i) = u(x_i)$

(ii) $$\text{sign } \sigma_i = \begin{cases} +1 & \text{if } h^*(x_i) = u(x_i) \\ -1 & \text{if } h^*(x_i) = \ell(x_i) \end{cases}$$

(iii) $$\int_a^b (f-h^*)h_j = \sum_{i=1}^{k} \sigma_i \, h_j \, (t_i) \; , \quad j = 1, \cdots, n \; .$$

(Here $h_1, \cdots, h_n$ is a basis for the set of approximations) .

Computational algorithms of the Remes type have been formulated, convergence proofs obtained, and numerical examples solved in [6], [8].

IV. Mixed Norm Approximation. In designing beam patterns for antenna arrays and in designing the frequency response of a digital filter, one may wish to use different criteria of approximation for different intervals. For example, one might use a uniform norm

$$\max_{x \text{ in } I_1} |f(x) - p(x)|$$

in the passband interval I_1 and a weighted least squares norm

$$\left(\int_{I_2} w(x) |f(x)-p(x)|^2 dx \right)^{1/2}$$

in the stopband interval I_2. Two possible approaches are:

$$\text{(i)} \quad \underset{p}{\text{minimize}} \int_{I_2} w(x) |f(x)-p(x)|^2$$

subject to $-\varepsilon \le f(x)-p(x) \le \varepsilon$ for all x in I_1;

and

$$\text{(ii)} \quad \text{minimize} \left\{ \lambda \max_{x \text{ in } I_1} |f(x)-p(x)|^2 + (1-\lambda) \int_{I_2} w(x) |f(x)-p(x)|^2 dx \right\}$$

where $0 < \lambda < 1$. The "mixed norm" problem (ii) can be converted to a quadratic programming problem, and solved; cf [2]. It is shown in [2] that, for $f(x)$ of a certain type, (i) and (ii) are

equivalent in the following sense: If $p_\lambda(x)$ solves the problem (ii) for some λ, $0 < \lambda < 1$, then there exists an $\varepsilon > 0$ such that $p_\lambda(x)$ solves problem (i). Conversely, if $p_\varepsilon(x)$ solves (i) for some ε, there exists λ, $0 < \lambda < 1$ such that $p_\varepsilon(x)$ solves (ii). It is also shown that mixed norm approximation is related to vectorial approximation, developed by A. Bacopoulos and others [15]. In [6] the "mixed norm" in (ii) above is generalized to $\lambda^r||f-p||_1^{\,r} + (1-\lambda)^r\,||f-p||_2^{\,r}$ where $1 \leq r$ and $||\ \ ||_1$, $||\ \ ||_2$ are two specified norms. Again an equivalence theorem is obtained. The "$r = \infty$" case, formulated as $\max\,\{\lambda||f-p||_1,\ (1-\lambda)||f-p||_2\}$, is studied carefully in [3] for $||f-p||_1$ a discrete uniform norm and $||f-p||_2$ a least square norm: a computational algorithm is formulated, proved to converge, and tested numerically. This algorithm is an analogue of an algorithm of C. L. Lawson, cf. [16].

V. Other Results. In this section we discuss other work partially supported by the Grant, namely [10] and [5]. Let $-\infty < a < 0 < b < \infty$, and f a real function, continuous on $[a, b]$ but not a polynomial there. Given integers k, n

denote

$$\text{(1)} \qquad E_n(f) = \min \max_{a\leq x\leq b} |f(x)-p_n(x)|$$

where the minimum is taken over polynomials of degree at most n. Let $E_n^k(f)$ denote the right hand side of (1) where the minimum is taken over all polynomials of degree at most n whose coefficient of x^k is 0. It follows from work in [17], [18] that if $f^{(k)}$ exists and satisfies a Lipschitz condition of order α $(0<\alpha<1)$ throughout $[a, b]$ and if

$f^{(k)}(0) \neq 0$ then there exists a positive constant A_n such that

$$(2) \quad \frac{E_n^{(k)}(f)}{E_n(f)} \geq A_k n^{\alpha} , \quad n = 0, 1, \cdots .$$

In [10] a converse theorem has been obtained which essentially states that if inequality (2) holds, then $f^{(k)}$ exists and satisfies a Lipschitz condition of order α on each closed interval $[a', b']$ with $a < a' < b' < b$. Hence a remarkable theorem is obtained identifying a certain smoothness of f with the growth of $\frac{E_n^k(f)}{E_n(f)}$.

Note that this theorem could be considered a "partial basis" result of type described in Section II.

In conclusion we mention briefly [5] where Muntz-type closure theorems are obtained for sequences of the form

$$\left\{w(t)t^{\alpha_k}\right\}_{k=1}^{\infty}$$ on unbounded intervals.

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